



پژوهشکده بیمه، به منظور کمک به گسترش دانش بیمهای، ترجمه کتاب

## **Risk Management in Banks and Insurance Companies**

را در دستور کار خود قرار داده است. لذا از کلیه اساتید، پژوهشگران، صاحبنظران و کارشناسان دعوت میشود که در صورت تمایل به ترجمه کتاب مذکور، کاربرگ درخواست ترجمه پیوست را به همراه سوابق علمی و اجرایی خود و ترجمه صفحات ذکر شده با ذکر عنوان کتاب، حداکثر تا تاریخ 1403/07/30 به آدرس ایمیل ir.ac.irc@nashr ارسال فرمایند.



**کاربرگ درخواست ترجمه کتاب** 





# **ب- سابقه تألیف/ترجمه )حداقل 3 عنوان از آثار خود را اعالم بفرمائید(**



# **ج- سابقه اجرایی**



One of the most important continuous probability distributions is the shaped. The appearance and properties of the normal distribution are deter-<br>mined-by two parameters: normal distribution. The density function of the normal distribution is bell-<br>shaped. The appearance and properties of the normal distribution are deterhalo *normal distribution*. The density function of the normal distribution is bell-

- Expected value  $\mu$ : It describes the number that the random variable assumes on average.
- Standard deviation  $\sigma$ : It shows the dispersion around the expected value.

integral from  $-\infty$  to  $\infty$ ), always has a value of one. The total area enclosed by the curve of the normal distribution (hence the

The *standard normal distribution* is a normal distribution in which the expected value  $= 0$  and the standard deviation  $= 1$ .

Often the standard deviation is not known and must therefore be estimated. In this case, the usage of the  $t$ -*Student distribution* is recommended, which has a fatter tail-end than the normal distribution. This indicates that values further away from the expected value are assumed to be more probable compared to the normal distribution. The appearance and properties of the t-Student distribution is determined by one parameter:

• The number of degrees of freedom n. The larger the number of degrees of freedom, the closer the t-Student distribution approximates the standard normal distribution.

In addition to the normal distribution and the t-Student distribution, there tion function best describes the data, the instrument of *calibration* is used. are a large number of other distribution functions. To find out which distribu-Statistical software or spreadsheet software is used to calibrate the empirical data to a theoretical distribution function. The distributions are fitted to the empirical data estimating the parameters for the distribution function. The software tools often also report the results of goodness-of-fit tests (for example, Smirnov goodness-of-fit test, Shapiro-Wilk test, or Anderson-Darling chi-square goodness-of-fit test, Box-Cox transformation, Kolmogorovgoodness-of-fit test). The estimated distribution parameters can then be used for risk assessment and modelling.

#### **Important Formulas**

Workbook: Case Study Risk Management Worksheet: Density & Dis-tribution Function

Calculation of the expected value, which corresponds to the mean value of the continuous returns:

**29 Volume 2: Modelling Volatilities** 39

The acronym ARCH stands for "Autoregressive Conditional Heteroscedasticity". ARCH models try to consider that volatilities follow a certain pattern. This property of changing volatilities over time is called heteroscedasticity. ARCH models are also *autoregressive*, i.e., volatility is measured as a function of its predecessor, i.e., it is *conditional*.

ARCH models are particularly well suited for short-term forecasts based on current historical data. Here, we consider data from the past five days for the ARCH model.

#### **Important Formulas**

Workbook: Case Study Risk Management Worksheet: ARCH

The formula for the variance  $\sigma^2$  at time t in the ARCH(*m*) model is as follows:

$$
\sigma_t^2 = \gamma \cdot V_L + \sum_{i=1}^m \alpha_i \cdot r_{n-1}^2
$$
 (2.19)

 $V_L$  = Long-term variance of the time series

 $\gamma$  = Weighting factor of  $V_L$ 

 $r_{n-i}^2$  = Squared return (= variance) on the previous day

 $\alpha_i$  = Weighting factor on day *i* 

 $m =$  Number of observations

 $n =$  Period of the estimator  $= m+1$ 

The ARCH(m) model implies that the variance  $\sigma_t^2$  can be explained using the average long-term variance and the weighted historical variance from  *observations.* 

Since the sum of the weights equals one:

$$
\gamma + \sum_{i=1}^{m} \alpha_i = 1 \tag{2.20}
$$

In the ARCH $(m)$  model, younger observations are assigned higher weights and older observations are assigned lower weights.

With  $\omega = \gamma \cdot V_L$  the above formula can also be written as follows:

The geometric Brownian Motion can be described analogously with the following differential equation:

$$
dS_t = S_t \cdot (\mu_S \cdot dt + \sigma_S \cdot dW_t) \tag{2.27}
$$

 $S_t$  = Value at time t

 $dS_t$  = Small change of the value

 $d =$  Differential

 $dt =$  Small time interval

Here,  $dS_t$  describes a small change in the share price in a short time interval dt. The change results from the influence of two components: the drift rate  $\mu_s$  and the volatility of the share  $\sigma_s$ . The drift rate  $\mu_s$  is the average expected return of a stock. If  $\mu_s > 0$ , the value of the share price increases in expectation, if  $\mu_s < 0$  the share price decreases in expectation. The volatility of the share  $\sigma_S$  controls the influence of chance, which is represented by the small change in the Wiener process, i.e. by  $dW_t$ . This differential Eq.  $(2.27)$  can be solved with the help of Itô's lemma (Merton, Robert C. (1976) Option pricing when underlying stock returns are discontinuous). The solution results in the representation in Eq.  $(2.26)$ .

Geometric Brownian Motion is based on the following assumptions:

 $W_o = 0$ 

- The paths are continuous
- The increments of the Wiener process are stochastically independent and nor-<br>mally distributed, i.e.  $W_t W_s \sim N(0, t s)$  for  $0 \le s \le t$ .

In the most popular financial mathematical models, Geometric Brownian Motion is used to model assets. Compare the Black-Scholes model for option pricing  $(Assignment 2.11)$  and the Merton model for calculating credit default probabilities  $(Assignment 2.17)$ .

A few practical tips:

- Although the paths are continuous, they are modelled discretely for simplicity .reasons
- Since Brownian Motion is a future view, the volatility expected by the market is often used.

To model Brownian Motion in Matlab/Excel, we use the following simple representation:

$$
S_t = S_0 L_1 L_2 \dots L_n \text{ with } L_i = \frac{S(t_i)}{S(t_{i-1})}
$$
 (2.28)

79 Course Unit 4: Derivation of Risk Ratios with the Help of Black-Scholes

#### Delta

The Delta is the first derivative of the Black-Scholes formula according to the option's strike price. It indicates how much the price of an option changes when the strike price rises/falls by one monetary unit.

The following applies to the Delta of a call  $C$  with share price  $S$ :

$$
\Delta_C = \frac{\partial C}{\partial S} = N(d_1) \ge 0
$$
\n(2.39)

and to the Delta of a put  $P$ :

$$
\Delta_P = \frac{\partial P}{\partial S} = N(d_1) - 1 \ge 0 \tag{2.40}
$$

 $S =$  Strike price

 $N =$  Distribution function of the standard normal distribution

 $d_1$  = Definition of the value see Assignment 2.11

#### Gamma

The Gamma is the second derivative after the share price. It is the only "Greek of second order", which is counted among the classical Greeks.

It is valid for the Gamma of a call  $C$  and for the Gamma of a put  $P$ :

$$
\Gamma_{C/P} = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}} \ge 0
$$
\n(2.41)

 $S_0$  = Share price at time 0

 $N'$  = Density function of the standard normal distribution

 $d_1$  = Definition of the value see Assignment 2.11

 $\sigma$  = Volatility

 $T = \text{Term of the option}$ 

#### Vega

Another factor influencing the Black-Scholes formula is volatility. Vega indicates the strength of the change in the option price with a change in the volatility of the underlying.

It is valid for the Vega of a call  $C$  and for the Vega of a put  $P$ :

$$
V_{C/P} = S_0 \sqrt{T} N'(d_1) \ge 0
$$
\n(2.42)

 $S_0$  = Share price at time 0

 $N'$  = Density function of the standard normal distribution

 $d_1$  = Definition of the value see Assignment 2.11

 $T =$  Term of the option

112 3 Credit Risks

### Assignment 20: Vasicek Model—Estimation of Parameters from Historical Data

### Task

Estimate the Probability of Default (PD) and the default correlation  $\rho$  based on historical default rates. For this, refer to the data of the annual global study on corporate defaults and rating changes of the rating agency  $S\&P$ . Assume a default correlation of 0.15 and a default probability of  $1\%$  as starting values.

#### **Content**

In the previous assignments, the Probability of Default  $(PD)$  and the default correlation were chosen notionally. In the next step, these parameters are estimated based on historical data using the Maximum Likelihood Method. The Maximum Likelihood Method calculates the values of the parameters that maximise the probability of occurrence of the historical values (here default rates). The parameters  $PD$  and  $\rho$  are determined in such a way that they best describe the observations that have occurred so far.

The process is as follows:

- 1. Selection of initial values for the Probability of Default and the default .correlation
- 2. Computation of the logarithm of the probability density of the Probability of Default  $g(x)$  based on historical data.
- 3. Calculation of the sum of these values.
- 4. Maximisation of this sum using the solver in Excel or the "fminsearch" command in Matlab to find the corresponding values of the probability of default (PD) and the default correlation.

#### **Important Formulas**

Workbook: Case Study Risk Management Worksheet: Vasicek Determination of the log-likelihood:

$$
g(x) = \ln\left(\sqrt{\frac{1-\rho}{\rho}}e^{\frac{1}{2}\left[\left(N^{-1}(x)\right)^2 - \left(\frac{\sqrt{1-\rho}N^{-1}(x)-N^{-1}(PD)}{\sqrt{\rho}}\right)^2}{\rho}}\right)\right)
$$
(3.19)

 $N^{-1}$  = Inverse of the standard normal distribution

 $PD =$  Probability of Default

 $\rho$  = Default correlation

However, it is questionable at what point the VaR model cannot be used beyond the predicted number of exceptions. Therefore, it must be checked whether the number of exceptions is sufficient to reject the assumptions for the  $VaR_{99\%}$  sufficiently. For this purpose, a test of significance known from statistics can be used.

#### **Important Formulas**

Workbook: Case Study Risk Management Worksheet: Backtesting

First, the available data is analysed. The data set contains the data from *n* considered days. In the example above  $n = 1.000$ . The number of times the Value at Risk is exceeded, the so-called exceptions  $c$  are determined from historical data. The probability that such an exception will occur is  $p$  and can be calculated by:

$$
p = \frac{c}{n} \tag{5.10}
$$

 $c =$  Number of times the Value at Risk is exceeded, so-called exceptions  $n =$  Number of data in the data set

Consequently, if the  $VaR_{1-\alpha}$  is true, the probability that an exception occurs in the data set should be equal to  $\alpha$ . Consequently, should hold:

$$
p = \alpha \tag{5.11}
$$

 $p =$  Probability for an exception in the data set

 $\alpha$  = Probability for an exception in the VaR model

This statement can be verified with a significance test. In order to perform a significance test, a level of significance must be selected. In practice,  $5\%$  is usually used for this purpose. The level of significance means that an error probability of  $5\%$ is accepted. It should be noted that in statistical tests a statement can only be rejected and cannot be confirmed.

If the number of observed exceptions  $c$  is greater than the number of expected exceptions  $\alpha \cdot n$ , it must be checked whether the Value at Risk underestimates the risk. The random variable  $X$  describes the number of exceptions. One must verify :whether

$$
P(X \ge c | p = \alpha) = 1 - P(X < c | p = \alpha) = 1 - P(X \le c - 1 | p = \alpha) < 5\% \tag{5.12}
$$

If the probability that  $c$  or more exceptions occur is less than the significance level (here  $5\%$ ), then the model is rejected. This means that it is unlikely that c or more exceptions can be observed.

If the number of observed exceptions is  $c$  is smaller than the number of expected exceptions  $\alpha \cdot n$ , one must test whether the Value at Risk overestimates the risk. One must verify whether:

V)

If the effect of the change in the market interest rate on the price of a bond shall be analysed, the price function of the bond is differentiated with respect to the market interest rate. The following relationship is obtained:

$$
\frac{dP(r)}{dr} = -\frac{1}{1+r} \cdot D \cdot P(r) \tag{5.19}
$$

dr<sup>1</sup>  $\frac{dP(r)}{dr}$  = First derivative of the price function according to the market interest rate  $dP(r)$  = Change in price in the event of a small change in the market interest rate  $dr =$  Small change in the market interest rate

Transforming this equation yields:

$$
\frac{dP}{P} = -\frac{1}{1+r} \cdot D \cdot dr = -MD \cdot dr \qquad (5.20)
$$

As a simplification, it is  $P(r) = P$ . The modified Macaulay Duration is the proportionality factor:

$$
MD = -\frac{1}{1+r} \cdot D \tag{5.21}
$$

Modified Duration is therefore nothing more than the first derivative of the present value function according to the interest rate divided by the price (present value) of the bond.

The concept of Convexity improves the results of the Duration. This is achieved by adding the second derivative in the Taylor expansion. The following relationship can be derived:

$$
P(r + \Delta r) \approx P(r) + \frac{dP}{dr} \cdot \Delta r + \frac{1}{2!} \cdot \frac{d^2 P}{dr^2} (\Delta r)^2
$$
 (5.22)

 $\Delta r =$  Change in the market interest rate

 $dr^2$  $d^2P$  $\equiv$  Second derivative of the price function according to the market interest rate

and thus:

$$
\frac{P(r + \Delta r) - P(r)}{P(r)} = \frac{\Delta P}{P} \approx -MD \cdot \Delta r + \frac{1}{2} \cdot C \cdot (\Delta r)^2 \tag{5.23}
$$

where  $C$  describes the Convexity: